$$z_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad z_2 = \begin{bmatrix} x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \qquad h_j = \sqrt{2}\,\sin\left(\frac{\left(j-\frac{1}{2}\right)\pi}{2n}\right)$$

} windowing function for window of length $2n$

Show that below holds true after windowing

$$\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \frac{1}{2}(NMz_1)_{n\ldots 2n-1} + \frac{1}{2}(NMz_2)_{0\ldots n-1}$$

Component multiplication (to apply windowing)

$z_1 * h$

$$z_1 = \begin{bmatrix} x_1h_1 \\ x_2h_2 \\ x_3h_3 \\ x_4h_4 \end{bmatrix}, \quad z_2 = \begin{bmatrix} x_3h_1 \\ x_4h_2 \\ x_5h_3 \\ x_6h_4 \end{bmatrix}$$

component multiplication (to undo windowing)

$$NMz_1 = \begin{bmatrix} x_1 - Rx_2 \\ -Rx_1 + x_2 \\ x_3 + Rx_4 \\ Rx_3 + x_4 \end{bmatrix} \rightarrow (NMz_1)*h = \begin{bmatrix} x_1h_1 - R(x_2h_2) \\ -R(x_1h_1) + x_2h_2 \\ x_3h_3 + R(x_4h_4) \\ R(x_3h_3) + x_4h_4 \end{bmatrix}$$

$$NMz_2 = \begin{bmatrix} x_3 - Rx_4 \\ -Rx_3 + x_4 \\ x_5 + Rx_6 \\ Rx_5 + x_6 \end{bmatrix} \rightarrow (NMz_2)*h = \begin{bmatrix} x_3h_1 - R(x_4h_2) \\ -R(x_3h_1) - x_4h_2 \\ x_5h_3 + R(x_6h_4) \\ R(x_5h_3) + x_6h_4 \end{bmatrix}$$

Proof of $\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \frac{1}{2}(NMZ_1) + \frac{1}{2}(NMZ_2)$

$\boxed{X_3}$

$$x_3 = \frac{1}{2}\left[(x_3h_3 + R(x_4h_4))(x_3h_3 - R(x_4h_4))\right]$$
$$+ \frac{1}{2}\left[(x_3h_1 - R(x_4h_2))(-R(x_3h_1) + x_4h_2)\right]$$

$$x_3 = \frac{1}{2}\left[x_3h_3^2 + h_3R(x_4h_4) + x_3h_1^2 + h_1R(x_4h_2)\right]$$

$$x_3 = \frac{1}{2}\left(x_3h_3^2 + x_3h_1^2\right)$$

$$x_3 = \frac{1}{2}x_3\left(h_3^2 + h_1^2\right)$$

$$h_1 = \sqrt{2}\,\sin\left(\frac{(1-1/2)(\pi)}{2n}\right)$$

$n \ldots 2n-1 =$

or 1 to $2n$

↳ $j = n+1 \ldots 2n$ for x_3

$$h_3 = \sqrt{2}\,\sin\left(\frac{(n+1-\frac{1}{2})\pi}{2n}\right)$$

$$h_3 = \sqrt{2}\,\sin\left(\frac{\pi n}{2n} + \frac{(1-1/2)\pi}{2n}\right)$$

$$h_3 = \sqrt{2}\,\sin\left(\frac{\pi}{2} + \frac{(1-1/2)\pi}{2n}\right)$$

$$h_3 = \sqrt{2}\,\cos\left(\frac{(1-1/2)\pi}{2n}\right)$$

$$x_3 = \frac{1}{2}x_3\left(h_3^2 + h_1^2 2\right) \qquad M = \frac{(1-1/2)\pi}{2n}$$

$$x_3 = \frac{1}{2}\left[2(\sin^2(M) + \cos^2(M))\right]x_3 = \frac{1}{2}(2(1)) = \boxed{x_3}\ ✓$$

$\boxed{X_4}$

$$x_4 = \frac{1}{2}\left[(x_4h_4 + R(x_3h_3))(x_3h_3 + R(x_4h_4))\right]$$
$$+ \frac{1}{2}\left[(x_3h_1 - R(x_4h_2))(-R(x_3h_1) + x_4h_2)\right]$$

↓

$$x_4 = \frac{1}{2}\left[x_4h_4^2 + h_4R(x_3h_3) - x_4h_2R\,x_3 + x_4h_2^2\right.$$

$$x_4 = \frac{1}{2}\left(x_4h_4^2 + x_4h_2^2\right)$$

$$x_4 = \frac{1}{2}x_4\left(h_4^2 + h_2^2\right)$$

$h_2 \Rightarrow \frac{n}{2}+1$ to n

↳
$$h_2 = \sqrt{2}\,\sin\left(\frac{\left(\frac{n+1}{2} - \frac{1}{2}\right)(\pi)}{2n}\right)$$

$h_4 \to \frac{3n}{2}+1$ to $2n$

↳
$$h_4 = \sqrt{2}\,\sin\left(\frac{\left(\frac{3n}{2}+1-\frac{1}{2}\right)(\pi)}{2n}\right)$$

$$h_4 = \sqrt{2}\,\sin\left(\frac{\pi}{2} + \frac{\left(\frac{n}{2}+1-\frac{1}{2}\right)(\pi)}{2n}\right)$$

$$N = \left(\frac{n}{2}+1\right)\frac{(\pi)}{2n}$$

$$\sin\left(\frac{\pi}{2} + N\right) = \cos(N)$$

$$h_4 = \sqrt{2}\,\cos(N)$$

$$x_4 = \frac{1}{2}x_4\left(h_2^2 + h_4^2\right)$$

$$x_4 = \frac{1}{2}x_4\left(2\cos^2(N) + 2\sin^2(N)\right) = \frac{1}{2}(x_4)(2) = x_4$$